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LINEAR, NONLINEAR SMALL-AMPLITUDE, STEADY AND SHOCK WAVES IN MAGNETICALLY STABILIZED LIQUID–SOLID AND GAS–SOLID FLUIDIZED BEDS

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Abstract—The propagation of solid concentration disturbances in fluidized beds in an external magnetic field is considered. Both solid particles and the liquid phase are assumed to be simultaneously magnetizable. The total fluid–particle interaction force is supposed to include the inertial component proportional to the relative acceleration of fluid and particles. The effect of simultaneous magnetization of particles and fluid as well as the influence of the inertial component of interphase interaction force on the resulting criterion of stability of the uniform fluidization are analysed. Consideration is given to the dispersion phenomena in the wave propagation process. The model of propagation of nonlinear waves is developed in approximation of small finite-amplitude waves. The basic equations are reduced to the Korteweg–de Vries–Burgers equation for the departure of solid concentration from the uniform state. Possible configurations of the concentration wavefront are studied, including the oscillating wavefronts and small-amplitude shocks. The conditions of realization of each possible configuration are obtained. The propagation of a long finite-amplitude nonlinear steady wave is considered. The conditions for the existence of the steady concentration wave are derived. The structure of the wavefront is studied and the thickness of the front is calculated as a function of magnetic and other physical parameters and the concentrations ahead of and behind the front. The conditions across the concentration shock in the fluidized bed of magnetic particles are obtained. The shock speed is calculated. The obtained results can be used to analyse structures of boundaries of bubbles, slugs and solid clusters formed in magnetically stabilized fluidized beds. In conclusion the analogy between the basic equations of magnetic fluidized beds and equations of the "particle bed" model by Foscolo & Gibilaro is briefly discussed in order to analyse the possibility to apply the developed approach to the study of the considered classes of nonlinear waves in conventional fluidized beds.

Key Words: two-phase flow, gas particulate flow, fluidization, magnetic stabilization, concentration waves, jump conditions

INTRODUCTION

A theoretical study of the propagation of linear and nonlinear concentration waves in magnetically stabilized fluidized beds is of fundamental interest giving information on the mechanisms of formation of concentration discontinuities, bubbles and slugs as well as on possible methods of stabilization of bubbling beds by external magnetic fields.

The theoretical analysis of the propagation of linear concentration waves given by Rosensweig (1979, 1985) for gas–solid fluidized beds has been later extended by Rosensweig & Cyprios (1991) for liquid–solid beds; the considerable development of theory has been achieved in the latter due to the consideration of fluidized beds of nonmagnetic particles in a magnetic fluid as well as beds of magnetic particles in a neutral fluid. It must be noted that the corrected expressions for magnetic forces following from the mean-field theory of magnetic field in a two-phase dispersion were used by Rosensweig & Cyprios (1991). However the stability criterion derived in the latter represents only the sufficient condition of linear stability.

The necessary and sufficient criterion of magnetic stabilization of liquid–solid fluidized beds of magnetic particles in a neutral fluid has been later obtained by Sergeev & Muromsky (1994).

The further development of the theoretical analysis of wave propagation in liquid–solid magnetic fluidized beds requires to use corrected (compared to the cited works) expression for the liquid–particle interaction force, including, in particular, the inertial component of interphase interaction. An interesting opportunity could also be given to the study of fluidized beds in the case of simultaneous magnetization of the liquid and solid phases.

The analysis of the simplest class of nonlinear concentration disturbances—long concentration waves—has been given by Sergeev & Muromsky (1994). The Burgers equation with the nonlinear dissipative term has been obtained and the qualitative description has been given for the mechanism of evolution of nonlinear concentration disturbances and to the thickness and amplitude of concentration shock fronts formed in fluidized beds. The details of the concentration (voidage) distribution at the wavefront has not been studied.

It can be expected that a certain progress in understanding of the wave propagation and evolution process and, henceforth, formation of bubbles and slugs and structures of their boundaries is associated with the analysis of different classes of nonlinear concentration waves in a fluidized bed. In particular, a simple description can be given to the nonlinear small-amplitude waves, steady waves and concentration shocks on the basis of general equations of magnetic fluidized beds.

Here we should note the mathematical analogy between the equations of momentum conservation of the liquid and solid phases of magnetic fluidized beds and the equations of the well known "particle bed" model of conventional fluidized beds proposed by Foscolo & Gibilaro (1984, 1987). The mathematical similarity enables to use some of the principle results obtained in the theory of magnetically stabilized bed for the analysis of propagation of nonlinear concentration waves within the framework of the "particle bed" model.

BASIC EQUATIONS AND CONSTITUTIVE RELATIONS

The propagation of one-dimensional disturbances of solid concentration in the vertical direction in a fluidized bed of magnetic particles in a magnetically neutral fluid and/or in a fluidized bed of magnetically neutral particles in a magnetic fluid is considered. The model of two-phase dispersion as a double continuum consisting of two mutually penetrating and interacting ideal fluids (referred to below as the Two-Fluid Model) is used. The general form of one-dimensional mass and momentum conservation equations is as follows:

$$\frac{\partial \varepsilon}{\partial t}+\frac{\partial(\varepsilon v_{\mathrm{f}})}{\partial z}=0 \qquad [1]$$

$$\frac{\partial \alpha}{\partial t}+\frac{\partial(\alpha v_{\mathrm{p}})}{\partial z}=0 \qquad [2]$$

$$\varepsilon+\alpha=1 \qquad [3]$$

$$\rho_{\mathrm{f}}\varepsilon\left(\frac{\partial v_{\mathrm{f}}}{\partial t}+v_{\mathrm{f}}\frac{\partial v_{\mathrm{f}}}{\partial z}\right)=-\frac{\partial p_{\mathrm{f}}}{\partial z}-\rho_{\mathrm{f}}\varepsilon \boldsymbol{g}+F_{\mathrm{I}}+f_{\mathrm{fm}} \qquad [4]$$

$$\rho_{\mathrm{p}}\alpha\left(\frac{\partial v_{\mathrm{p}}}{\partial t}+v_{\mathrm{p}}\frac{\partial v_{\mathrm{p}}}{\partial z}\right)=-\frac{\partial p_{\mathrm{p}}}{\partial z}-\rho_{\mathrm{p}}\alpha \boldsymbol{g}-F_{\mathrm{I}}+f_{\mathrm{pm}} \qquad [5]$$

where ε is the void fraction, α the volumetric concentration of solid phase, v_{f} and v_{p} are the interstitial fluid velocity and the mean velocity of solid particles, z is the vertical coordinate, ρ_{f} and ρ_{p} are the fluid and solid densities respectively, p_{f} is the fluid pressure, p_{p} is the effective pressure of the solid phase, F_{I} the interphase interaction force, f_{fm} and f_{pm} are the magnetic forces exerted on the fluid and particles.

Like in Sergeev & Muromsky (1994), solid particles are assumed to be spherical and to have equal diameter d_{p} such that the Reynolds number $\mathrm{Re}=Ud_{\mathrm{p}}/\nu \ll 1$, where U is the superficial undisturbed fluid velocity, ν kinematic viscosity of the fluid. Unlike the cited work, the total interphase interaction force F_{I} is assumed to contain the additional "inertial" component related to the relative acceleration of the solid and liquid phases. With these assumptions the interphase interaction force can be written in the form (see, for example, Jackson 1971):

$$F_{\mathrm{I}}=\alpha\frac{\partial p_{\mathrm{f}}}{\partial z}-18\frac{\rho_{\mathrm{f}}\nu}{d_{\mathrm{p}}^{2}}\alpha\Phi(\varepsilon)u-\rho_{\mathrm{f}}C(\varepsilon)\frac{\mathrm{d}u}{\mathrm{d}t} \qquad [6]$$

where

$$u = v_f - v_p \quad [7]$$

is the relative velocity of phases; the Richardson–Zaki (1954) form

$$\Phi(\varepsilon) = \varepsilon^{-n} \quad (n = 2.8) \quad [8]$$

is used for the viscous drag function; the effective added mass coefficient for the suspension $C(\varepsilon)$ is often used in the form

$$C = \frac{\varepsilon}{2}. \quad [9]$$

The substantial derivative in the inertial component of the interaction force [6] can be written as (see, for example, Jackson 1971)

$$\frac{du}{dt} = \left(\frac{\partial v_f}{\partial t} + v_f \frac{\partial v_f}{\partial z}\right) - \left(\frac{\partial v_p}{\partial t} + v_p \frac{\partial v_p}{\partial z}\right). \quad [10]$$

Using the assumptions described in Sergeev & Muromsky (1994), the constitutive relations for the magnetic forces can be written in the Kelvin form (see, for example, Rosensweig 1985; Rosensweig & Cyprios 1991)

$$f_{fm} = \mu_0 \varepsilon M_f(H_f) \frac{\partial H_f}{\partial z} \quad [11]$$

$$f_{pm} = \mu_0 \alpha M_p(H_p) \frac{\partial H_p}{\partial z} \quad [12]$$

where μ_0 is the magnetic permeability of vacuum, H_f and H_p are the average local values of the magnetic field strength in the liquid and solid phases respectively and $M_f(H_f)$ and $M_p(H_p)$ are the magnetizations of phases.

The hydrodynamic equations [1]–[5] should be considered together with the equations of magnetic field:

$$H + M = B_0/\mu_0, \quad M = \varepsilon M_f + \alpha M_p, \quad H = \varepsilon H_f + \alpha H_p \quad [13a]$$

$$M_f = \chi_f(H_f)H_f, \quad M_p = \chi_p(H_p)H_p, \quad M = \chi H \quad [13b]$$

where B_0 is the magnetic induction of the uniform external field, H and M are respectively the average local magnetic field strength and magnetization of two-phase dispersion and χ_p, χ_f and χ are the chord magnetic susceptibilities of liquid phase, solid particles and two-phase dispersion respectively. The relationship between susceptibilities is given by the generalization of the Clausius–Mosotti formula following from the mean-field theory (see Landauer 1978) in the form

$$\frac{\chi - \chi_f}{\chi + 2\chi_f + 3} = \alpha \frac{\chi_p - \chi_f}{\chi_p + 2\chi_f + 3}. \quad [14]$$

To close the system of equations and constitutive relationships [1]–[14], like in Sergeev & Muromsky (1994), we assume below (with the exception of the section dedicated to the steady waves), the equation of state of the solid phase is in the form $p_p = 0$ as a matter of convenience due to the lack of a suitably validated constitutive law relating particle pressure in suspensions of magnetic particles. It was already noted by Sergeev & Muromsky (1994) that the effective solid pressure can be neglected if the magnetization of solid particles satisfies the inequality

$$\mu_0 M_p^2 \gg \rho_p U^2. \quad [15]$$

Introducing the dimensionless variables

$$z^* = \frac{z}{L}, \quad t^* = \frac{U}{L}t, \quad v_f^* = \frac{v_f}{U}, \quad v_p^* = \frac{v_p}{U}, \quad p_f^* = \frac{p_f}{\rho_f U^2} \quad [16]$$

where L is the linear scale of disturbances, the dimensionless mass and momentum conservation equations can be reduced to (the superscript * is henceforth omitted)

$$\varepsilon_t + v_f \varepsilon_z + \varepsilon (v_f)_z = 0 \qquad [17]$$

$$\varepsilon_t + v_p \varepsilon_z - (1 - \varepsilon)(v_p)_z = 0 \qquad [18]$$

$$[1 + \mathrm{De}\, \varepsilon^{-1} C(\varepsilon)]\{(v_p)_t + v_p (v_p)_z\} - \mathrm{De}[1 + \varepsilon^{-1} C(\varepsilon)]\{(v_f)_t + v_f (v_f)_z\} = -(1 - \mathrm{De})\mathrm{Fr} + \kappa\, \mathrm{Fr}\, \varepsilon^{-n-1}(v_f - v_p) + \gamma(\varepsilon)\varepsilon_z \qquad [19]$$

where the density (De), Froude (Fr) and Stokes (κ) numbers are given by

$$\mathrm{De} = \frac{\rho_f}{\rho_p}, \quad \mathrm{Fr} = \frac{gL}{U^2}, \quad \kappa = \frac{18\rho_f \nu U}{\rho_p g d_p^2} \qquad [20]$$

and the "magnetic force" parameter γ in [19] is the following:

$$\gamma = \frac{B_0^2}{\mu_0 \rho_p U^2} T(\varepsilon) \qquad [21]$$

where

$$T(\varepsilon) = \frac{\mu_0^2}{B_0^2}\left(M_p(H_p)\frac{dH_p}{d\varepsilon} - M_f(H_f)\frac{dH_f}{d\varepsilon} \right). \qquad [22]$$

The strength of magnetic field within the solid and liquid phases as a function of the local voidage should be found from the equations of magnetic field [13] and the Clausius–Mosotti-type formula [14] providing that the functions $\chi_f(H_f)$ and $\chi_p(H_p)$ are known. Henceforth, the parameter γ can be found as a function of the local voidage: $\gamma = \gamma(\varepsilon)$. The last relation closes the system of [17]–[19].

The parameter γ can be explicitly expressed as a function of ε in the following cases:

1. Linear magnetization of phases: $B_0/\mu_0 \ll M_{fs}$, $B_0/\mu_0 \ll M_{ps}$ (here and below the subscript "s" denotes the magnetic saturation). The magnetization of solid particles and fluid can be considered as linear with respect to magnetic field strength, so that χ_f = constant and χ_p = constant. From [13] and [14] we have:

$$T(\varepsilon) = \frac{2(\chi_f - \chi_p)^2(9\chi_f^2 + \chi_p^2 + 5\chi_f\chi_p + 18\chi_f + 6\chi_p + 8)}{(\chi_f + 1)^2\{3(\chi_p + 1) + 2(\chi_f - \chi_p)\varepsilon\}^3}. \qquad [23]$$

It can be easily seen from [23] that $\gamma \geqslant 0$; $\gamma = 0$ as $\chi_f = \chi_p$, so that the effects created by the magnetic properties of fluid can compensate the effects of magnetization of solid particles.

In the case of nonmagnetic fluid ($\chi_f = 0$), [23] reduces to

$$T(\varepsilon) = \frac{2\chi_p^2(\chi_p + 2)(\chi_p + 4)}{[3 + \chi_p(1 + 2\alpha)]^3}. \qquad [24]$$

When solid particles are magnetically neutral ($\chi_p = 0$), [23] yields:

$$T(\varepsilon) = \frac{2\chi_f^2(3\chi_f + 2)(3\chi_f + 4)}{(\chi_f + 1)^2(3 + 2\chi_f\varepsilon)^3}. \qquad [25]$$

2. Magnetically saturated bed: $B_0/\mu_0 \gg M_{fs}$, $B_0/\mu_0 \gg M_{ps}$. In the case of a strong magnetic field, both the fluid and solid particles are magnetically saturated so that $M_f = M_{fs}$ = constant and $M_p = M_{ps}$ = constant. From [13] and [14] we find:

$$T = \frac{2\mu_0^2}{3B_0^2}(M_{fs} - M_{ps})^2. \qquad [26]$$

In the case of nonmagnetic fluid, [26] reduces to (see Sergeev & Muromsky 1994):

$$T = \frac{2\mu_0^2 M_{ps}^2}{3B_0^2}. \qquad [27]$$

For the fluidized bed of nonmagnetic particles in the magnetic fluid [26] yields:

$$T = \frac{2\mu_0^2 M_{fs}^2}{3B_0^2}. \qquad [28]$$

3. $M_{ps} \ll B_0/\mu_0 \ll M_{fs}$. In this case the solid phase is magnetically saturated, while the linear magnetization of the fluid can be assumed, so that $\chi_f = \text{constant}$, $M_p = M_{ps} = \text{constant}$ and T can be found in the form:

$$T = \frac{2\mu_0^2}{B_0^2} \times \frac{M_{ps}(\chi_f + 1) - \mu_0^{-1} B_0 \chi_f}{(\chi_f + 1)^2(3 + 2\chi_f\varepsilon)^3} \times \left\{[3(\chi_f + 1)(2\chi_f + 3) - 2\chi_f^2\alpha](\chi_f + 1)M_{ps} - (2\chi_f + 3)^2\chi_f \frac{B_0}{\mu_0}\right\}. \quad [29]$$

It must be noted that $T < 0$ ($\gamma < 0$) if

$$1 > \frac{\chi_f + 1}{\chi_f} \times \frac{\mu_0^2 M_{ps}}{B_0} > \left\{1 + \frac{\chi_f(3 + 2\chi_f\varepsilon)^3}{(2\chi_f + 3)^2}\right\}^{-1}. \quad [30]$$

From the results obtained in the next section it follows that when [30] holds, the magnetic field has a destabilizing effect on the fluidized bed.

The expression for γ can also be obtained in case of magnetically saturated fluid and linear magnetization of solid particles ($M_{fs} \ll B_0/\mu_0 \ll M_{ps}$), i.e. when $\chi_p = \text{constant}$ and $M_f = M_{fs} = \text{constant}$. Since the form of this expression is rather complicated and it is not used below, we omit it here.

LINEAR WAVES AND STABILITY (SIMULTANEOUS MAGNETIZATION OF FLUID AND SOLID PARTICLES; EFFECT OF INERTIAL COMPONENT OF INTERPARTICLE INTERACTION)

The stability criterion of the uniform fluidized bed of magnetic particles in the nonmagnetic fluid has been obtained by Sergeev & Muromsky (1994) in the case when the inertial component of the interphase interaction force is neglected ($C = 0$).

The parameters of the uniform steady state can be found as the following solution of [17]–[19]:

$$\varepsilon_0^{n+1} = \frac{\kappa}{1 - \text{De}}, \quad v_f = v_0 = \frac{1}{\varepsilon_0}, \quad v_p = 0. \quad [31]$$

We note that the first formula in [31] gives the estimation for the value of κ.

Linearizing [17]–[19] in the vicinity of the steady state [31] we obtain the following equation for the disturbance of voidage $\eta = \varepsilon_0 - \varepsilon$ in the form

$$\xi\left(\frac{\partial}{\partial t} + c_1\frac{\partial}{\partial z}\right)\left(\frac{\partial}{\partial t} + c_2\frac{\partial}{\partial z}\right)\eta + \left(\frac{\partial}{\partial t} + c_0\frac{\partial}{\partial z}\right)\eta = 0 \quad [32]$$

where

$$\xi = \frac{\varepsilon_0^{n+1}(\varepsilon_0 + \text{De}(\alpha_0 + E_0))}{\kappa\ \text{Fr}}, \quad E(\varepsilon) = \frac{C(\varepsilon)}{\varepsilon}, \quad c_0 = \frac{(n+2)\alpha_0}{\varepsilon_0} \quad [33]$$

c_0 is the velocity of the kinematic concentration wave at $\varepsilon = \varepsilon_0$, c_1 and c_2 are the characteristic velocities of the higher order:

$$c_{1,2} = \frac{\alpha_0\varepsilon_0}{\varepsilon_0 + \text{De}(\alpha_0 + E_0)}\left\{\frac{\text{De}(1 + E_0)}{\varepsilon_0^2} \pm \left[\frac{\varepsilon_0 + \text{De}(\alpha_0 + E_0)}{\alpha_0\varepsilon_0}\gamma - \frac{\text{De}(1 + E_0)(1 + \text{De}\,E_0)}{\alpha_0\varepsilon_0^3}\right]^{1/2}\right\}. \quad [34]$$

The steady state is linearly stable while (Witham 1974)

$$c_2 < c_0 < c_1. \quad [35]$$

Incorporating [34], it can be easily shown that [35] reduces to the stability criterion

$$\gamma_0 > \varepsilon^{-3}\Omega_*(\varepsilon_0; \text{De}) \quad [\gamma_0 = \gamma(\varepsilon_0)] \quad [36]$$

where

$$\Omega_*(\varepsilon;\mathrm{De}) = \frac{\alpha[\varepsilon(n+2)(1-\mathrm{De}) + \mathrm{De}(n+1)(1+E)]^2 + \varepsilon\,\mathrm{De}(1+E)(1+\mathrm{De}\,E)}{\varepsilon(1-\mathrm{De}) + \mathrm{De}(1+E)}. \qquad [37]$$

It must be noted that the stability criterion obtained by Sergeev & Muromsky (1994), in the case when the inertial component of the total interphase interaction force is neglected, i.e. $C = 0$ ($E = 0$), has been written in the form being incorrect at $\mathrm{De} \neq 0$.

The linear stability criteria for fluidized beds of magnetic particles in a nonmagnetic fluid and nonmagnetic particles in a magnetic fluid have been obtained by Rosensweig & Cyprios (1991). It was already discussed by Sergeev & Muromsky (1994) that the mentioned criteria represent the necessary conditions of linear stabilization, while [37] gives both necessary and sufficient conditions.

From the steady state momentum conservation equations it follows that the undisturbed superficial fluid velocity U is the function of voidage in the uniform fluidized bed:

$$U = \frac{g d_p^2(\rho_p - \rho_f)\varepsilon_0^{n+2}}{18\rho_f \nu} \qquad [38]$$

so that the dimensionless function $\gamma(\varepsilon)$ defined by [21] can be written in the form

$$\gamma(\varepsilon) = \sigma T(\varepsilon)\varepsilon^{-2n-4} \qquad [39]$$

where the dimensionless parameter σ depending only on physical and mechanical properties of phases and the strength of the external magnetic field is given by

$$\sigma = \frac{324\rho_f^2\nu^2 B_0^2}{\mu_0 g^2 d_p^4 \rho_p(\rho_p - \rho_f)^2} \qquad [40]$$

and the dimensionless function $T(\varepsilon)$ is defined in [22]. Incorporating [39] we find the stability criterion in the form

$$\sigma > \frac{\Omega(\varepsilon_0;\mathrm{De})}{T(\varepsilon_0)} \qquad [41]$$

where

$$\Omega = \varepsilon^{2n+1}\Omega_*. \qquad [42]$$

$\Omega_*(\varepsilon_0;\mathrm{De})$ is given by [37].

The explicit form of the stability criterion can be obtained in the following cases:

1. Linear magnetization of phases: $B_0/\mu_0 \ll M_{fs}$, $B_0/\mu_0 \ll M_{ps}$. The function $T(\varepsilon)$ is given by [23]. In cases of magnetic particles in a nonmagnetic fluid and nonmagnetic particles in a magnetic fluid, $T(\varepsilon)$ reduces to [24] and [25] respectively.

2. Magnetically saturated bed: $B_0/\mu_0 \gg M_{fs}$, $B_0/\mu_0 \gg M_{ps}$. In this case T does not depend on ε. Incorporating [26] the stability criterion can be written in the form

$$\sigma_* > \Omega(\varepsilon;\mathrm{De}) \qquad [43]$$

where

$$\sigma_* = \frac{216\mu_0\rho_f^2\nu^2(M_{ps} - M_{fs})^2}{g^2 d_p^4 \rho_p(\rho_p - \rho_f)^2}. \qquad [44]$$

The stability criterion [43] is illustrated in figure 1 for $\mathrm{De} = 0.36$ by the solid line for the function $C(\varepsilon)$ given by [9] and by the dashed line for $C = 0$ (the detailed view for high solid concentrations, i.e. voidages close to ε_{mf} are given in figure 1(b).

It must be noted that the accounting of the inertial effects in the interphase interaction force shows the decreasing of the interval of parameters corresponding to the stabilization of the uniform state as $C(\varepsilon) \not\equiv 0$. The qualitative analysis of the stability criterion for $C = 0$ in case of nonmagnetic fluid is given by Sergeev & Muromsky (1994).

3. Magnetic saturation of solid particles, linear magnetization of fluid: $M_{ps} \ll B_0/\mu_0 \ll M_{fs}$. The function $T(\varepsilon)$ in [41] is now given by [29]. If $\chi_f = 0$, [41], incorporating the function $T(\varepsilon)$ in the

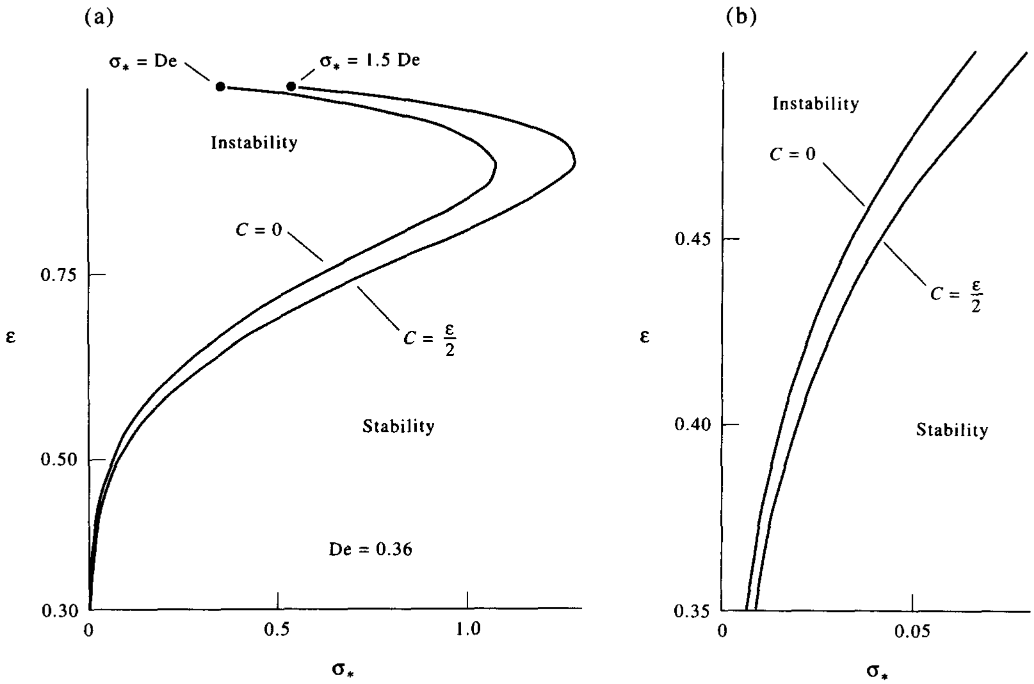

Figure 1. (a) Linear stability criterion. (b) Detailed view for voidages close to ε_{mf}.

form [29], coincides with [43] and [44] at $M_{fs} = 0$. It has been already noted that the destabilizing effect is possible in case *3* if [30] is valid.

Although the stability analysis can be given entirely on the basis of [32] using Witham's approach, as it has been done above, to analyse some other features of the propagation of linear waves, such as dispersion phenomena and the behaviour of the disturbances reflected from the upper boundary of the bed, we consider the dispersion equation corresponding to the linearized system of [17]–[19]:

$$\xi[\omega^2 - (c_1 + c_2)\omega k + c_1 c_2 k^2] + i(\omega - c_0 k) = 0 \qquad [45]$$

where k is the wavenumber, ω the frequency.

To simplify the further analysis we consider the case of high Froude numbers such that

$$\kappa \,\mathrm{Fr} \gg 1 \qquad [46]$$

so that ξ given by [34] is a small parameter. We should underline here that [46] is typical for a wide range of fluidized beds of small particles.

To analyse the dispersion relation following from [45] it is convenient to represent the function $\omega(k)$ in the form of expansion in terms of the small parameter ξ. We assume that the magnetic field strength and magnetization of phases are not very high so that $\gamma \ll \xi^{-2}$ (hence $|c_{1,2}| \ll \xi^{-1}$); we also take into account that the principle contribution to the Fourier series of disturbances is given by the components corresponding to wave numbers $k \simeq O(1)$. From [45] it follows that the dispersion relation $\omega = \omega(k)$ consists of two branches; the first one corresponds to the waves propagating upwards (in the direction of the undisturbed gas flow):

$$\omega = k\{c_0 + i\xi k\mu_1 + (\xi k)^2\mu_2 + O[(\xi k)^3]\} \qquad [47]$$

where

$$\mu_1 = (c_0 - c_1)(c_0 - c_2) = \frac{\alpha_0 \varepsilon_0}{\varepsilon_0 + \mathrm{De}(\alpha_0 + E_0)}(\Omega_* - \gamma) \qquad [48]$$

$$\mu_2 = \mu_1(2c_0 - c_1 - c_2) = 2\alpha_0\varepsilon_0^{-1}\mu_1\left\{n + 2 + \frac{\mathrm{De}(1 + E_0)}{\varepsilon_0 + \mathrm{De}(\alpha_0 + E_0)}\right\} \quad [49]$$

and $\Omega_* = \Omega_*(\varepsilon_0; \mathrm{De})$ is given by [37]. It must be noted that the stability criterion in the form [36] follows already from the "long-wave" approximation of dispersion relation given by [47] (indeed, the condition of stability $\mathrm{Im}\,\omega < 0$ immediately leads to [36]).

The second branch corresponds to the waves propagating downwards (for example, to the waves reflected from the upper boundary of the bed). The expansion of this branch in terms of ξ starts from the term $O(\xi^{-1})$:

$$\omega = -i\frac{1}{\xi} + O(1) = -i\frac{\kappa\,\mathrm{Fr}}{\varepsilon_0^{n+1}\{\varepsilon_0 + \mathrm{De}(\alpha_0 + E_0)\}} + O(1) \quad [50]$$

so that the waves propagating downwards rapidly damp as $\exp(-t/\xi)$. This means that for large Froude numbers the waves reflected from the upper boundary can be withdrawn from consideration.

NONLINEAR SMALL-AMPLITUDE CONCENTRATION WAVES

If the uniform state of a fluidized bed is unstable, the structure of disturbances cannot be described in the general case. Nevertheless, long small-amplitude nonlinear waves allow rather simple description in the considered model.

The following procedure has been proposed by Sagdeev (1964) for the analysis of small-amplitude nonlinear waves in rarefied plasma and later applied to the concentration waves in fluidized beds by Golo & Myasnikov (1975) in case of noninteracting solid particles and by Kurdyumov & Sergeev (1987) and Sergeev (1988) in case of interacting particles.

From the sum of mass conservation equations [1] and [2], we find the following relationship between the velocities of phases and the voidage:

$$\varepsilon v_f + \alpha v_p = 1 \quad [51]$$

where the constant in the RHS of [51] is found from the assumption on the undisturbed flow far from the disturbance (as $z \to \infty$).

Expressing the fluid velocity through the velocity of the particle phase from [51] as

$$v_f = \varepsilon^{-1}(1 - \alpha v_p) \quad [52]$$

the combined momentum equation [19], can be written in the form

$$v_p = v_p^0(\varepsilon) + \gamma(\kappa\,\mathrm{Fr})^{-1}\varepsilon^{n+2}\varepsilon_z - (\kappa\,\mathrm{Fr})^{-1}\varepsilon^{n+2}\{[1 + \mathrm{De}\,\varepsilon^{-1}C(\varepsilon)][(v_p)_t + v_p(v_p)_z]$$
$$- \mathrm{De}\,[1 + \varepsilon^{-1}C(\varepsilon)][(v_f)_t + v_f(v_f)_z]\} \quad [53]$$

where

$$v_p^0(\varepsilon) = 1 - \frac{1 - \mathrm{De}}{\kappa}\varepsilon^{n+2} \quad [54]$$

To study small-amplitude nonlinear waves we assume the parameter $(\kappa\,\mathrm{Fr})^{-1}$ to be small (i.e. $\xi \ll 1$, where ξ is given by [34]), so that the long waves are considered, and the external magnetic field and magnetizations of phases such that $\gamma \ll \kappa\,\mathrm{Fr}$.

Using [53] together with [52], we approximate v_p as a function of ε by successive iterations to the accuracy of $O[(\kappa\,\mathrm{Fr})^{-2}] = O(\xi^2)$; this procedure corresponds to the branch of dispersion [47], i.e. to the waves propagating upwards without damping. Substituting the obtained iterative expressions for v_p into the mass conservation equation [18] yields

$$\varepsilon_t + c(\varepsilon)\varepsilon_z = Q \quad [55]$$

where $c(\varepsilon)$ is the kinematic wave velocity:

$$c = 1 + \kappa^{-1}(1 - \mathrm{De})\varepsilon^{n+1}[n + 2 - (n + 3)\varepsilon] \quad [56]$$

(Sergeev 1985), Q is the sum of the dissipative and dispersive terms of the orders ξ and ξ^2 respectively. It should be noted that the kinematic wave velocity in the uniform state c_0 can be obtained in the form [33] from [56], taking into account the first relation [31] between the parameters of the uniform state.

Neglecting Q the model considered by Sergeev (1985) can be obtained. In such a model the solutions are simple kinematic waves.

For finite small-amplitude waves at $\xi \ll 1$ it can be supposed that an effect of dispersion and dissipation is small, that the solution in the form of a quasisimple wave (see Karpman 1973; Ablowitz & Segur 1981) can be considered:

$$v_p = v_p^0(\varepsilon) + \psi(z, t) \quad [57]$$

where $\psi = \mathrm{O}(\xi)$. The waves propagating downwards can be neglected due to their rapid damping.

Considering the small departure of the voidage from the uniform state:

$$\varepsilon = \varepsilon_0 - \eta \quad (\alpha = \alpha_0 + \eta), \quad \eta \ll 1 \quad [58]$$

linearizing the RHS of [55] and using the iterations to substitute at $t = \mathrm{O}(\xi^{-1})$ the space derivatives in Q instead of time derivatives, we obtain for small-amplitude nonlinear concentration disturbances the Korteweg–de Vries–Burgers equation:

$$\eta_t + (c_0 + \beta\eta)\eta_z + \xi\mu_1\eta_{zz} - \xi^2\mu_2\eta_{zzz} = 0 \quad [59]$$

where μ_1 and μ_2 are defined by [48] and [49] respectively, and

$$\beta = -\left(\frac{\mathrm{d}c}{\mathrm{d}\varepsilon}\right)_{\varepsilon=\varepsilon_0} = \frac{(n + 2)[(n + 3)\varepsilon_0 - (n + 1)]}{\varepsilon_0^2}. \quad [60]$$

The evolution of nonlinear concentration disturbances can now be analysed on the basis of [59] by the known methods (see, for example, Ablowitz & Segur 1981; Karpman 1973).

It can be assumed that an initial concentration disturbance is a simple wave described by the equation

$$\varepsilon_t + c(\varepsilon)\varepsilon_z = 0. \quad [61]$$

The evolution of the wave described by [61] indicates the formation of concentration discontinuities (Sergeev 1985; Fanucci *et al.* 1979; Kluwick 1983; Liu 1983; Needham & Merkin 1983). In the process of formation of the concentration jump the dissipative and dispersive phenomena start to play a role stabilizing a profile of the wavefront (Witham 1974); finally a formation of the wavefront with sharp but continuous change of voidage (solid concentration), i.e. concentration shock front, can be expected. Small-amplitude nonlinear waves can lead to the formation of shock fronts in the case of unstable uniform states of a fluidized bed as well as in case of stable states.

The profile of the shock front can be studied with the help of a steady wave solution [59], $\eta = \eta(z - Dt)$, where D is the propagation speed. It must be noted that such a solution is a quasisteady solution of [17]–[19] for the approximation of $\mathrm{O}(\xi^2)$. The well known methods developed for *KdVB* equation [59] (see Karpman 1973, Ablowitz & Segur 1981) give the following results.

The voidage distribution at the wavefront is oscillating as $\mu_c > \mu_1$, $\mathrm{Im}\,\mu_c = 0$ and monotonous as $\mu_c < \mu_1$, $\mathrm{Im}\,\mu_c = 0$, where

$$\mu_c = \sqrt{2\mu_2\beta(\varepsilon_- - \varepsilon_+)} = \frac{1}{\varepsilon_0}\sqrt{2\mu_2(\varepsilon_- - \varepsilon_+)(n + 2)[(n + 3)\varepsilon_0 - (n + 1)]} \quad [62]$$

ε_- and ε_+ are the voidages ahead and behind the wavefront, respectively.

We start with the unstable uniform state when $\gamma_0 < \Omega_*$ where Ω_* is given by [37]. From [48] it follows that in the unstable steady state $\mu_1 > 0$. It is easily seen from [49] that μ_2 and μ_1 are always of the same sign so that $\mu_2 > 0$. The rarefaction concentration shock front ($\varepsilon_+ > \varepsilon_-, \alpha_+ < \alpha_-$) occurs as

$$\varepsilon_-, \quad \varepsilon_+ < \varepsilon_* = (n+1)/(n+3) \simeq 0.655 \qquad [63]$$

while the compression shock front, ($\varepsilon_+ < \varepsilon_-$), occurs as

$$\varepsilon_-, \quad \varepsilon_+ > \varepsilon_*. \qquad [64]$$

The oscillations of voidage (solid concentration) appear in both the cases behind the shock front. We must note here that within the considered approximation of not very short waves ($\xi \ll 1$), the question on the existence of rarefaction shocks cannot be discussed.

In the case of the stable uniform state ($\gamma_0 > \Omega_*$) the rarefaction shock front is possible as ε_-, $\varepsilon_+ > \varepsilon_*$, while the compression front—as ε_-, $\varepsilon_+ < \varepsilon_*$; like in the case of the unstable steady state, voidage oscillations appear behind the shock front.

An analysis of the propagation speed of the shock is given in one of the following sections.

The above results can be used for qualitative analysis of structure of boundaries of bubbles and slugs in a fluidized bed.

FINITE-AMPLITUDE STEADY CONCENTRATION WAVES

As in the above section, we consider long waves such that [46] is valid. Now we suppose that the magnetic field strength and magnetizations of phases are high, such that $\gamma \geqslant O(\kappa\ \mathrm{Fr})$. From [53] and [59] it can be concluded that in this case the effects of dissipation and dispersion produced by hydrodynamical phenomena are negligibly small compared to the dissipation produced by the magnetic interaction in the two-phase dispersion.

Using the above assumptions, the last term in the RHS of [53] can be neglected, so that the combined momentum conservation equation, instead of as in [53], can be written in the form

$$v_p = v_p^0(\varepsilon) + (\kappa\ \mathrm{Fr})^{-1}\gamma(\varepsilon)\varepsilon^{n+2}\varepsilon_z \qquad [65]$$

where $v_p^0(\varepsilon)$ is given by [54]. Substituting [65] into the mass conservation equation, we obtain the following equation of propagation of the long-amplitude wave:

$$\varepsilon_t + c(\varepsilon)\varepsilon_z = m_L[\varepsilon^{n+2}(1-\varepsilon)T(\varepsilon)\varepsilon_z]_z \qquad [66]$$

where we introduced the new dimensionless parameter

$$m_L = \frac{d_p B_0^2}{18\mu_0 \rho_f \nu U L}. \qquad [67]$$

The dimensionless function $T(\varepsilon)$ in [66] is given by [22] in the general case, and by [23]–[29] in the particular cases considered in the first section.

In the particular case of magnetically saturated solid particles in the nonmagnetic fluid, when T, given by [27], does not depend on ε, [66] has been obtained and analysed by Sergeev & Muromsky (1994). The general qualitative description of the formation of concentration shock fronts has been given for small-amplitude nonlinear long waves; a structure of the wavefront, especially in case of a large amplitude, was not analysed in detail.

To analyse the structure of the wavefront, we consider the propagation of nonlinear steady concentration wave. Introducing the "steady wave variable"

$$Z = z - Dt \qquad [68]$$

the equation of propagation of steady concentration wave can be written in the form

$$[c(\varepsilon) - D]\varepsilon_Z = m_L[\varepsilon^{n+2}(1-\varepsilon)T(\varepsilon)\varepsilon_Z]_Z \qquad [69]$$

where now $\varepsilon = \varepsilon(Z)$. We consider the usual steady wave boundary conditions:

$$\varepsilon = \varepsilon_- \quad \text{as} \quad Z \to +\infty$$
$$\varepsilon = \varepsilon_+ \quad \text{as} \quad Z \to -\infty. \qquad [70]$$

From [69] and B.C. [70] it follows that the propagation speed of the steady wave is:

$$D = \frac{1}{[\varepsilon]} \int_{\varepsilon_-}^{\varepsilon_+} c(\varepsilon)\,d\varepsilon = 1 + \frac{1-\mathrm{De}}{\kappa}\,\delta(\varepsilon_+, \varepsilon_-) \quad [71]$$

where

$$\delta = \frac{[\alpha\varepsilon^{n+2}]}{[\varepsilon]}. \quad [72]$$

Here and below the square brackets are used to denote the jump of a parameter across the wavefront:

$$[A] = A_+ - A_-. \quad [73]$$

It can be seen from [71] that the speed of the steady wave approaches the kinematic wave velocity as the wave amplitude decreases ($[\varepsilon] \to 0$).

The solution of the problem [69], [70] can be found in the implicit form

$$Z = \frac{m_L \kappa}{1-\mathrm{De}} \int^{\varepsilon} \frac{\alpha\varepsilon^{n+2} T(\varepsilon)\,d\varepsilon}{\psi(\varepsilon; \varepsilon_+, \varepsilon_-)} \quad [74]$$

where we indicate only the upper limit in the integral; the lower limit is an arbitrary value of voidage within the interval $(\varepsilon_+, \varepsilon_-)$ associated with the choice of the coordinate $Z = 0$; the function ψ in [74] is defined as

$$\psi(\varepsilon; \varepsilon_+, \varepsilon_-) = \frac{\kappa}{1-\mathrm{De}} \left\{ \int_{\varepsilon_-}^{\varepsilon} c(\varepsilon)\,d\varepsilon - D(\varepsilon - \varepsilon_-) \right\} = (\alpha\varepsilon^{n+2} - \delta\varepsilon) - (\alpha_-\varepsilon_-^{n+2} - \delta\varepsilon_-). \quad [75]$$

The function $\psi(\varepsilon)$ depends on ε_+ and ε_- as parameters and does not depend on physical properties of phases and other parameters of magnetic fluidized bed. The function ψ is zero at $\varepsilon = \varepsilon_-$ and $\varepsilon = \varepsilon_+$:

$$\psi(\varepsilon_+; \varepsilon_+, \varepsilon_-) = \psi(\varepsilon_-; \varepsilon_+; \varepsilon_-) = 0. \quad [76]$$

If the function $\psi(\varepsilon)$ does not have another zero between ε_- and ε_+ the steady wave exists, and from [74] it follows that the wave profile is monotonous.

Since the kinematic wave velocity has the form shown in figure 2, where ε_* given by [63] corresponds to the maximum of $c(\varepsilon)$, from [75] and [71] it follows that $\psi(\varepsilon)$ does not have another zero between ε_- and ε_+ and, henceforth, the steady wave exists if

$$\varepsilon_- \quad \text{and} \quad \varepsilon_+ > \varepsilon_* \quad [77a]$$

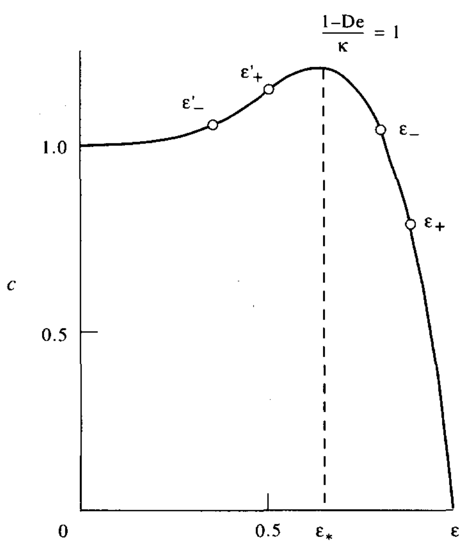

Figure 2. Kinematic wave velocity; ε_- and ε_+—void fractions in front of and behind the wavefront corresponding to the inequality [77a]; ε'_- and ε'_+ correspond to the inequality [77b].

or

$$\varepsilon_- \text{ and } \varepsilon_+ < \varepsilon_*. \qquad [77b]$$

The inequalities [77a] and [77b] give the sufficient conditions for the existence of the steady wave. If [77a] or [77b] are valid, the function $\psi(\varepsilon)$ has one of two forms, illustrated in figure 3(a).

The situation becomes more complicated when ε_+ and ε_- lie on different (with respect to ε_*) sides of the curve $c(\varepsilon)$, i.e. when

$$\varepsilon_+ < \varepsilon_* < \varepsilon_- \quad \text{or} \quad \varepsilon_- < \varepsilon_* < \varepsilon_+. \qquad [78]$$

The analysis of the RHS of [75] shows that when [78] is valid, $\psi(\varepsilon)$ can have four possible forms in dependence on the relation between ε_+ and ε_-, two are shown in figure 3(a) and another two are illustrated in figure 3(b). In the second case the function $\psi(\varepsilon)$ is zero at $\varepsilon = \varepsilon_c$ within the interval $(\varepsilon_+, \varepsilon_-)$, so that the solution in the form of a steady wave does not exist. It is seen from figure 3(b) that this form of the curve can exist only in the case when the derivatives of $\psi(\varepsilon)$ are of the same sign at $\varepsilon = \varepsilon_+$ and $\varepsilon = \varepsilon_-$. Henceforth, the necessary and sufficient condition for nonexistence of a steady concentration wave can be written as follows:

$$\psi'(\varepsilon_+; \varepsilon_+, \varepsilon_-) > 0, \quad \psi'(\varepsilon_-; \varepsilon_+, \varepsilon_-) > 0 \qquad [79a]$$

or

$$\psi'(\varepsilon_+; \varepsilon_+, \varepsilon_-) < 0, \quad \psi'(\varepsilon_-; \varepsilon_+, \varepsilon_-) < 0. \qquad [79b]$$

Introducing the functions

$$\Lambda_+(\varepsilon_+, \varepsilon_-) = [\varepsilon]\psi'(\varepsilon_+; \varepsilon_+, \varepsilon_-) \qquad [80a]$$

$$\Lambda_-(\varepsilon_+, \varepsilon_-) = [\varepsilon]\psi'(\varepsilon_-; \varepsilon_+, \varepsilon_-). \qquad [80b]$$

from [75] and [72] we find:

$$\Lambda_+ = \alpha_- \varepsilon_-^{n+2} + \varepsilon_+^{n+2}((n+1) - (n+2)\varepsilon_+) - \varepsilon_+^{n+1}\varepsilon_-((n+2) - (n+3)\varepsilon_+) \qquad [81a]$$

$$\Lambda_- = -\alpha_+ \varepsilon_+^{n+2} - \varepsilon_-^{n+2}((n+1) - (n+2)\varepsilon_-) + \varepsilon_-^{n+1}\varepsilon_+((n+2) - (n+3)\varepsilon_-). \qquad [81b]$$

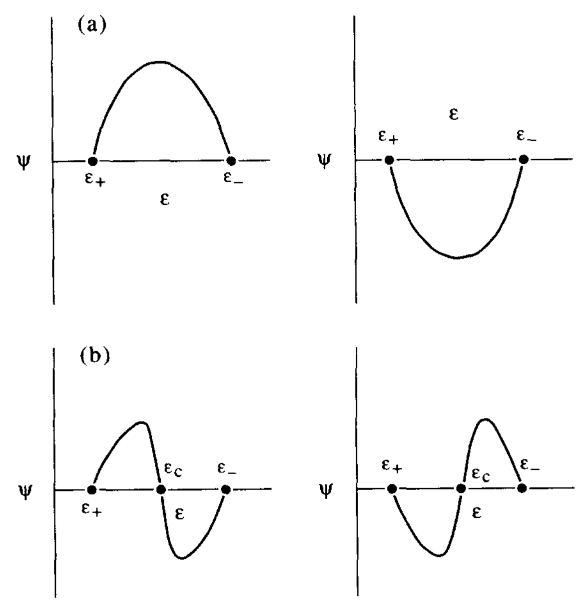

Figure 3. Sketch of possible behaviour of function $\psi(\varepsilon)$ within the interval $(\varepsilon_+, \varepsilon_-)$.

In the plane $(\varepsilon_+, \varepsilon_-)$ the boundaries of the domain within which the steady wave does not exist are given by the algebraic equations:

$$\Lambda_+(\varepsilon_+, \varepsilon_-) = 0, \quad \Lambda_-(\varepsilon_+, \varepsilon_-) = 0. \qquad [82]$$

The domain of nonexistence of the steady wave obtained by the numerical solution of the system [82] is shown in figure 4.

It must be noted that besides the lines drawn in figure 4 [82] has the trivial solution $\varepsilon_+ = \varepsilon_-$; such a solution must be withdrawn from the consideration since Λ_+ and Λ_- do not change their signs across the line $\varepsilon_+ = \varepsilon_-$ (it can be easily checked that $\partial\Lambda_+/\partial\varepsilon = \partial\Lambda_-/\partial\varepsilon = 0$ at $\varepsilon = \varepsilon_+ = \varepsilon_-$).

Now we analyse the structure of the steady wave in case when [77a] and [77b] are valid. To simplify the further consideration we suppose that both solid and liquid phases are magnetically saturated (including the case when one of the phases is magnetically saturated, and the other is nonmagnetic), so that $T = \text{constant}$ is given by [26]. Since $\psi = 0$ only at $\varepsilon = \varepsilon_+$ and $\varepsilon = \varepsilon_-$, separating singularities we represent the integrant in [74] in the form

$$\frac{\alpha\varepsilon^{n+2}}{\psi(\varepsilon; \varepsilon_+, \varepsilon_-)} = \frac{\alpha_-\varepsilon_-^{n+2}[\varepsilon]}{\Lambda_-(\varepsilon_+, \varepsilon_-)} \times \frac{1}{\varepsilon - \varepsilon_-} + \frac{\alpha_+\varepsilon_+^{n+2}[\varepsilon]}{\Lambda_+(\varepsilon_+, \varepsilon_-)} \times \frac{1}{\varepsilon - \varepsilon_+} + R(\varepsilon; \varepsilon_+, \varepsilon_-) \qquad [83]$$

where the function R is regular within $[\varepsilon_+, \varepsilon_-]$. The solution [74] can now be written as

$$Z = -\lambda_+(\varepsilon_+, \varepsilon_-)\ln|\varepsilon - \varepsilon_+| + \lambda_-(\varepsilon_+, \varepsilon_-)\ln|\varepsilon - \varepsilon_-| + \frac{m_L \kappa T}{1 - \mathrm{De}} \int_{\varepsilon_-}^{\varepsilon} R(\varepsilon; \varepsilon_+, \varepsilon_-)\, d\varepsilon \qquad [84]$$

where

$$\lambda_+ = \frac{m_L \kappa T}{1 - \mathrm{De}} \times \frac{\alpha_+\varepsilon_+^{n+2}[\varepsilon]}{|\Lambda_+|}, \quad \lambda_- = \frac{m_L \kappa T}{1 - \mathrm{De}} \times \frac{\alpha_-\varepsilon_-^{n+2}[\varepsilon]}{|\Lambda_-|} \qquad [85]$$

λ_+ and λ_- can be understood as the characteristic lengths of the "exponential parts" of the wave profile when ε approaches respectively ε_+ and ε_-.

The thickness of the wavefront can now be estimated as

$$L_Z = \lambda_+ + \lambda_- + L_R \qquad [86]$$

where L_R is the thickness of the "regular" part of the wavefront given by the last integral term in [84]. The direct calculation of $R(\varepsilon)$ from [83] shows that when the inequalities [77a] and [77b] are valid, the function $R(\varepsilon; \varepsilon_+, \varepsilon_-)$ does not differ very much from the linear function within the

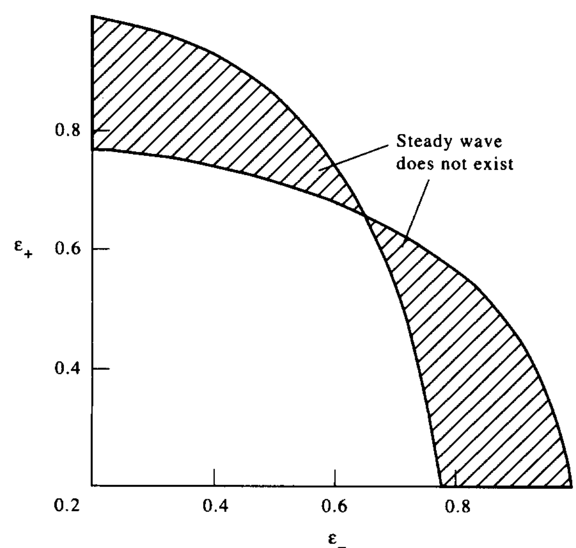

Figure 4. The domain of nonexistence of the steady wave (voidages lower than 0.2 are not illustrated).

interval $[\varepsilon_+, \varepsilon_-]$. Hence, the thickness of the regular part of the wavefront can be safely estimated as

$$L_R = \frac{m_L \kappa T}{1-\mathrm{De}}\left|\int_{\varepsilon_-}^{\varepsilon_+} R\,\mathrm{d}\varepsilon\right| \simeq \frac{m_L \kappa T}{1-\mathrm{De}} \times \tfrac{1}{2}\,|R(\varepsilon_+;\varepsilon_+,\varepsilon_-) + R(\varepsilon_-;\varepsilon_+,\varepsilon_-)|. \quad [87]$$

From the representation [83] it can be easily shown that

$$R_+ + R_- = \frac{[\alpha\varepsilon^{n+2}\Lambda]}{\Lambda_+\Lambda_-}. \quad [88]$$

To estimate the thickness of the wavefront it is now convenient to return to the dimensional parameters. The total dimensional thickness of the wavefront L_Z^* can be written as

$$L_Z^* = L_0 P(\varepsilon_+, \varepsilon_-) \quad [89]$$

where the length L_0 can be found from [20], [26] and [67] as depending only on the physical properties of both phases:

$$L_0 = \frac{m_L \kappa T}{1-\mathrm{De}} L = \frac{2}{3}\frac{\mu_0 (M_{ps} - M_{fs})^2}{g d_p(\rho_p - \rho_f)} \quad [90]$$

and the function $P(\varepsilon_+, \varepsilon_-)$ follows from [85]–[88] in the form:

$$P = |[\varepsilon]|\left(\frac{\alpha_+\varepsilon_+^{n+2}}{|\Lambda_+|} + \frac{\alpha_-\varepsilon_-^{n+2}}{|\Lambda_-|}\right) + \frac{1}{2}\left|\frac{[\alpha\varepsilon^{n+2}\Lambda]}{\Lambda_+\Lambda_-}\right|. \quad [91]$$

The function $P(\varepsilon_+, \varepsilon_-)$ is shown in figure 5, where figures a, b, c and d illustrate the behaviour of P when ε_+ and ε_- are situated in different parts of $(\varepsilon_+, \varepsilon_-)$-plane with respect to the domain of nonexistence of the steady wave shown in figure 4. It can be seen from figure 5 that the thickness of the wavefront increases dramatically with approaching the boundaries of the domain of nonexistence of the steady wave. Besides, the thickness of the wavefront tends to infinity when $\varepsilon_+ \to \varepsilon_-$. The last phenomenon can be easily understood since, within the considered long wave approximation, as $\varepsilon_+ \to \varepsilon_- (=\varepsilon_0)$ the steady wave becomes a simple kinematic wave analysed by Sergeev (1985), Fanucci *et al.* (1979), Kluwick (1983), Liu (1983) and Needham & Merkin (1983). Since there is no shock formation in the propagation of the infinitely small-amplitude kinematic wave at $\varepsilon = \varepsilon_0$, the dimensionless length scale (i.e. the thickness of the wavefront) is of the order of $\kappa\,\mathrm{Fr} \gg 1$. Because we have supposed that $\gamma \simeq O(\kappa\,\mathrm{Fr})$ and omitted the terms of the orders higher than O(1) with respect to the small parameter $(\kappa\,\mathrm{Fr})^{-1}$ from [66], we find the infinite thickness of the steady wavefront at $\varepsilon_+ = \varepsilon_-$.

In the case when one or both phases are not magnetically saturated, the generalization of the formulae [89]–[91] can be represented as follows:

$$L_Z^* = \frac{B_0^2}{\mu_0(\rho_p - \rho_f) g d_p}\left\{|[\varepsilon]|\left(\frac{\alpha_+\varepsilon_+^{n+2}T(\varepsilon_+)}{|\Lambda_+|} + \frac{\alpha_-\varepsilon_-^{n+2}T(\varepsilon_-)}{|\Lambda_-|}\right) + \frac{1}{2}\left|\frac{[\alpha\varepsilon^{n+2}\Lambda T]}{\Lambda_+\Lambda_-}\right|\right\} \quad [92]$$

where one of the formulae for $T(\varepsilon)$ valid for a nonsaturated bed, i.e. [23]–[25] and [29], can be applied.

To conclude this section we consider briefly the case when the solid concentration ahead of the wave is zero, $\varepsilon_- = 1$. Such a situation corresponds, for example, to the upper boundary of a slug in a fluidized bed. In this case from [74] it follows that

$$\left.\frac{\mathrm{d}\varepsilon}{\mathrm{d}Z}\right|_{\varepsilon=\varepsilon_-=1} = \infty \quad [93]$$

and a weak concentration discontinuity appears at the front of the wave. The steady wave has a structure shown in figure 6. In this case, from [85] we have $\lambda_- = 0$.

CONCENTRATION SHOCK WAVES IN A FLUIDIZED BED OF MAGNETIC PARTICLES

Below we analyse the propagation of plane concentration discontinuity (concentration shock wave). The dimensional parameters will be used in the following section.

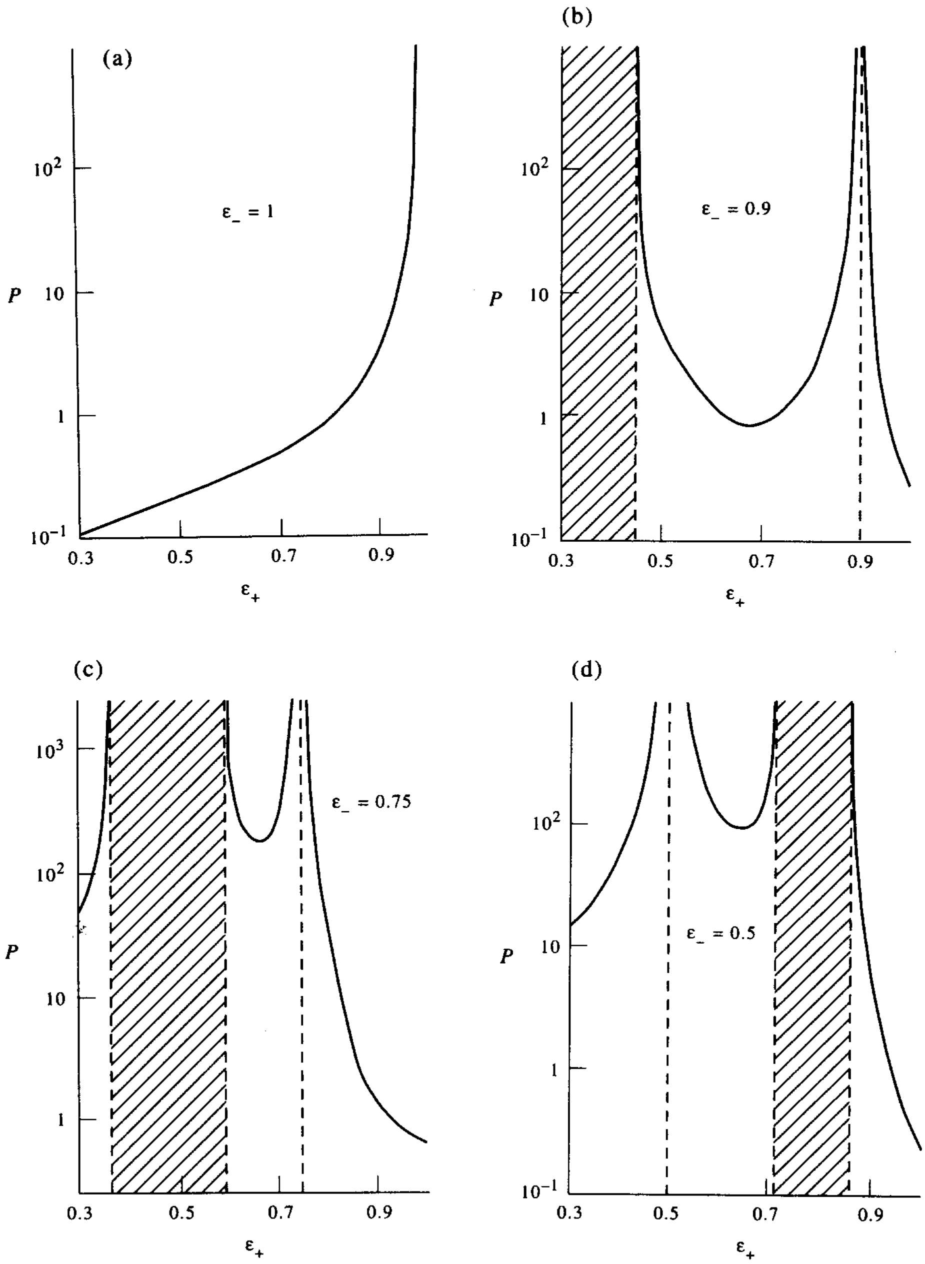

Figure 5. $P(\varepsilon_+, \varepsilon_-)$ as a function of the voidage behind the wavefront ε_+ at fixed ε_-. Steady wave does not exist within the dashed domains.

To simplify the following analysis we consider the case when only solid particles are magnetizable and $M_f = 0$.

Shock conditions

To obtain the shock conditions from the continuity and momentum conservation equations and the constitutive relations [6]–[14], the Kotchine's theory (1929) can be directly applied.

The conditions of mass conservation for the particle and fluid phases, respectively, are

$$[\varepsilon V_f] = 0, \quad [\alpha V_p] = 0 \qquad [94]$$

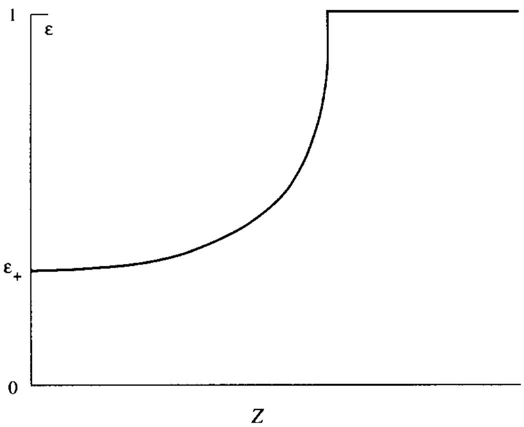

Figure 6. Structure of the steady wave at $\varepsilon_- = 1$.

where V_f and V_p are, respectively, the fluid and particle velocities in the coordinate system associated with the shock:

$$V_f = v_f - D, \quad V_p = v_p - D, \qquad [95]$$

we remind that the square brackets are used to denote the jump of value across the shock, see [73].

For the purposes of this section it is convenient to represent the conditions of momentum conservation of phases in the form of the condition of conservation of the combined momentum:

$$\rho_f[\varepsilon V_f^2] + \rho_p[\alpha V_p^2] + [p_f] - \mu_0 \int_{\alpha_-}^{\alpha_+} \alpha M_p \frac{dH_p}{d\alpha}\, d\alpha = 0 \qquad [96]$$

and the condition of conservation of the momentum of the fluid phase:

$$\rho_f[\varepsilon V_f^2] + \int_{\varepsilon_-}^{\varepsilon_+} \varepsilon \frac{dp_f}{d\varepsilon}\, d\varepsilon = 0. \qquad [97]$$

It is worth noting that [96] can be interpreted as the condition of conservation of the total momentum flux tensor across the shock. In the general three-dimensional case when both particles and the fluid phase are magnetizable this tensor has the form:

$$\Pi = \left(p_f - \mu_0 \int_{\alpha_-}^{\alpha_+} \alpha M_p \frac{dH_p}{d\alpha}\, d\alpha - \mu_0 \int_{\varepsilon_-}^{\varepsilon_+} \varepsilon M_f \frac{dH_f}{d\varepsilon}\, d\varepsilon\right)\mathbf{I} + \rho_p \alpha\, \|(v_p)_i (v_p)_j\| + + \rho_f \varepsilon\, \|(v_f)_i (v_f)_j\| \qquad [98]$$

where $(v_p)_k$ and $(v_f)_k$ are the components of vectors, $\mathbf{I}$ is the unit tensor ($I_{ij} = \delta_{ij}$ where δ_{ij} is the Kronecker delta), $\|v_i v_j\|$ the tensor with the components $v_i v_j$.

The functions $M_p(H_p)$ and $H_p(\alpha)$ in [96] (as well as $M_f(H_f)$ and $H_f(\varepsilon)$ in [98]) must be found from the system of equations and constitutive relations for the magnetic parameters [13] and [14].

The system of shock conditions [94]–[97] is not, however, closed. To close this system an additional constitutive relationship for the fluid pressure p_f, as a function of voidage ε valid at the shock surface, is required in order to calculate the integral in [97]. In principle, such a relation can be obtained based on the detailed study of the structure of the shock considered as a thin transitional layer of continuous change of the voidage and flow parameters. The integral in [97] can be represented in the form

$$\int_{\varepsilon_-}^{\varepsilon_+} \varepsilon \frac{dp_f}{d\varepsilon}\, d\varepsilon = \int_{-\Delta}^{+\Delta} \varepsilon \frac{\partial p_f}{\partial z}\, dz \qquad [99]$$

where 2Δ is the thickness of the transitional layer (it is natural to assume that Δ is of the order of particle size d_p). To calculate the integral [99] an additional model of the structure of transitional layer (in particular, the voidage distribution within the layer) is required. The development of such a model is, however, beyond the purpose of this paper.

The system of shock conditions can be closed without the detailed consideration of the structure of the shock in case of gas–solid dispersion. Since $\rho_f \ll \rho_p$, neglecting the first term in [97] we find

$$\int_{-\Delta}^{+\Delta} \varepsilon \frac{\partial p_f}{\partial z} \,\mathrm{d}z = 0. \qquad [100]$$

It is natural to assume the monotonous change of ε and, henceforth, p_f and other flow parameters within the transitional layer, so that we immediately obtain

$$[p_f] = 0. \qquad [101]$$

The last condition can be derived, without any additional assumption regarding the structure of the transitional layer, directly from the equation of momentum conservation of the fluid phase. As $\rho_f \ll \rho_p$ and $f_{fm} = 0$, [4] reduces to

$$\varepsilon \frac{\partial p_f}{\partial z} + 18 \frac{\rho_f \nu}{d_p^2} \alpha \Phi(\varepsilon) u = 0. \qquad [102]$$

Applying Kotchine's theory to [102] readily yields [101].

As soon as the integral in [96] is calculated, the system of shock conditions [94]–[96] and [101] for the gas–solid dispersion becomes closed.

For strong magnetic fields, when the solid phase is magnetically saturated so that $M_p = M_{ps} = \text{constant}$ from [13] and [14], we have

$$\frac{\mathrm{d}H_p}{\mathrm{d}\alpha} = -\tfrac{2}{3} M_{ps} \qquad [103]$$

and the integral in [96] becomes:

$$-\mu_0 \int_{\alpha_-}^{\alpha_+} \alpha M_p \frac{\mathrm{d}H_p}{\mathrm{d}\alpha} \,\mathrm{d}\alpha = \tfrac{1}{3} \mu_0 M_{ps}^2 (\alpha_- + \alpha_+)[\alpha]. \qquad [104]$$

In case of a relatively "weak" magnetic field when the magnetization of solid material is linear with respect to the magnetic field, i.e. $M_p = \chi_p H_p$ where $\chi_p = \text{constant}$, from [13] and [14] we have:

$$\frac{\mathrm{d}H_p}{\mathrm{d}\alpha} \,\mathrm{d}\alpha = -\frac{6B_0}{\mu_0} \times \frac{\chi_p}{W^2(\alpha)} \qquad [105]$$

and the integral in [96] becomes:

$$-\mu_0 \int_{\alpha_-}^{\alpha_+} \alpha M_p \frac{\mathrm{d}H_p}{\mathrm{d}\alpha} \,\mathrm{d}\alpha = \frac{36 B_0^2 \chi_p^3 \alpha_- \alpha_+ [\alpha]}{\mu_0 W^2(\alpha_-) W^2(\alpha_+)} \qquad [106]$$

where

$$W(\alpha) = \chi_p (1 + 2\alpha) + 3. \qquad [107]$$

Shock speed, speed of "sound"

To calculate the finite-amplitude shock speed we consider the gas–solid fluidized bed, so that $\rho_f \ll \rho_p$. In the case of the magnetically saturated suspension from [94]–[96], [101] and [104] the speed of the finite-amplitude concentration shock D_g follows in the form:

$$D_g^2 = \tfrac{1}{3} \mu_0 M_{ps}^2 \frac{\alpha_+ (\alpha_+ + \alpha_-)}{\rho_p \alpha_-}. \qquad [108]$$

The speed of "sound" in the two-phase gas–solid dispersion c_{sg} can be obtained from [108] as a speed of an infinitesimal shock:

$$c_{sg} = M_{ps} \sqrt{\frac{2\mu_0 \alpha}{3\rho_p}}. \qquad [109]$$

For the relatively weak magnetic field when the magnetization is linear with respect to the field strength, from [94]–[96], [101] and [106] we obtain:

$$D_g = \pm \frac{6B_0 \chi_p \alpha_+}{W(\alpha_-)\,W(\alpha_+)} \sqrt{\frac{\chi_p}{\mu_0 \rho_p}} \qquad [110]$$

$$c_{sg} = \frac{6B_0 \chi_p \alpha}{W^2(\alpha)} \sqrt{\frac{\chi_p}{\mu_0 \rho_p}} \qquad [111]$$

where $W(\alpha)$ is given by [107].

It must be underlined that there are two "speeds of sound" in a magnetic gas–solid fluidized bed: c_{sg}, corresponding to the wave driven by the stress created by the magnetic field, and c_0, kinematic wave velocity determined by the third relationship in [33]. While the long waves, as well as steady waves considered in the previous sections, always propagate upwards (long waves propagating downwards damp very rapidly), the shock waves considered in this section can propagate both upwards and downwards. It can be easily understood from [32], since the kinematic wave velocity is the lower order characteristic velocity, while the speed of "sound" given by [109] or [111] corresponds to the characteristic velocities of the higher order ($c_1 = -c_2$ for the gas–solid fluidized bed).

Below we call waves propagating upwards as "right" and downwards as "left" waves respectively; the signs, plus and minus, in [110] correspond to the "right" and "left" waves.

Since in the framework of the considered model the shock conditions cannot be formulated in the closed form for liquid–solid suspensions, to estimate the role of inertia of the fluid phase in the shock propagation process we consider below the propagation of weak shocks. The propagation speed of the weak shock is equal to the characteristic velocity of the higher order $c_{1,2}$ given by [34]; the signs, plus and minus, in [34] correspond to the "right" and "left" waves.

For the purpose of this section it is convenient to express the propagation speed of the shock in the liquid–solid fluidized bed through the speed of the weak shock in the gas–solid dispersion at the same values of solid concentration and magnetic parameters of the particle phase. We note that the magnetic parameter γ can be expressed from [34] through the speed of the weak shock (speed of "sound") in the gas–solid fluidized bed ($\rho_f \ll \rho_p$ so that $\mathrm{De} \ll 1$) as follows:

$$\gamma = \frac{c_{sg}^2}{\alpha U^2}. \qquad [112]$$

Incorporating [112], from [21] and [24] or [27] the expressions [109] and [111] can be easily obtained. At $C = \varepsilon/2$, incorporating [112] into [35], we obtain the expressions respectively for the "right" and "left" speeds of the weak shock (i.e. "right" and "left" speeds of "sound") in the liquid–solid suspension

$$(c_{sl})_+ = c_1, \quad (c_{sl})_- = c_2 \qquad [113]$$

through the speed of "sound" in the gas–solid suspension. The "right" and "left" speeds of "sound" for the suspension of glass particles in a water ($\mathrm{De} = 0.36$) are illustrated in figure 7.

We need to note here that while the "left" and "right" shock waves in the gas–solid fluidized bed (as $\rho_f \ll \rho_p$) propagate with the equal speed, the effect of the motion of the heavy liquid through the array of particles (i.e. at $\rho_f = O(\rho_p)$) leads to the pronounced difference in "left" and "right" speeds.

REMARK: ON NONLINEAR WAVES IN THE PARTICLE BED MODEL BY FOSCOLO GIBILARO

The basic equations of the widely known "Particle bed" model of a conventional (magnetically neutral) fluidized bed, proposed and developed by Foscolo & Gibilaro (1984, 1987), show a close

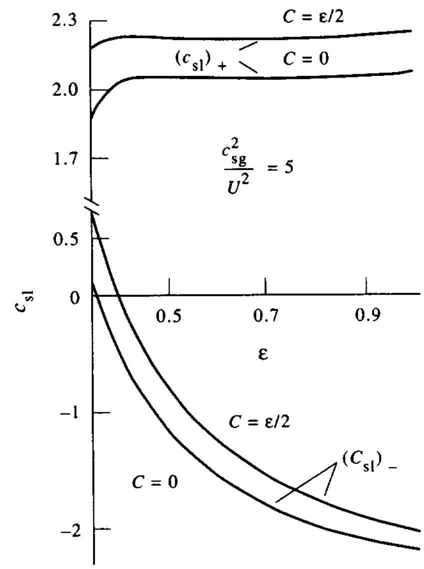

Figure 7. Speed of the weak shock (speed of "sound") in the liquid–solid fluidized bed as function of voidage; $(c_{sl})_+$ and $(c_{sl})_-$—"right" and "left" speeds respectively.

similarity to the considered equations of a fluidized bed with both magnetizable phases. The main feature of the "particle bed" model is the term

$$3.2gd_p(\rho_p - \rho_f)\alpha\frac{\partial\varepsilon}{\partial z} \qquad [114]$$

which appears with the opposite signs in the momentum conservation equations for both phases. The force [114] is considered by Foscolo & Gibilaro as an additional component of the fluid–particle interaction force due to the effect of solid concentration gradient in the bed.

Since the magnetic forces [11] and [12] in the model of the present work can be expressed as

$$f_{fm} = m_f(\varepsilon)\frac{\partial\varepsilon}{\partial z}, \quad f_{pm} = m_p(\varepsilon)\frac{\partial\varepsilon}{\partial z} \qquad [115]$$

where

$$m_f(\varepsilon) = \mu_0\varepsilon M_f(\varepsilon)\frac{dH_f}{d\varepsilon}, \quad m_p(\varepsilon) = \mu_0\alpha M_p(\varepsilon)\frac{dH_p}{d\varepsilon} \qquad [116]$$

and $M_f(\varepsilon)$, $H_f(\varepsilon)$, $M_p(\varepsilon)$, and $H_p(\varepsilon)$ can be found as function of ε, as a solution of the closed system of equations [13] and [14] (the equations of magnetic field with the Clausius–Mosotti relation between the magnetic parameters and the solid concentration), the formal similarity of the basic equations of both models becomes obvious.

The basic equations of the model by Foscolo & Gibilaro can be reduced to the system of partial differential equations [17]–[19] (at $C = 0$) for v_p, v_f and ε used in the present work with the dimensionless function $\gamma(\varepsilon)$ defined as

$$\gamma(\varepsilon) = \frac{3.2gd_p(1 - De)}{U^2\varepsilon}. \qquad [117]$$

This formal analogy enables to expect that the classes of nonlinear concentration waves considered in the present work, i.e. nonlinear small-amplitude waves, steady waves and concentration shock waves can be analysed within the framework of the Foscolo & Gibilaro model. It must be noted that the propagation of linear waves and stability of the uniform state were analysed in detail by Foscolo & Gibilaro (1987). Based on the "particle bed" model, the analysis of propagation of concentration shock waves in a conventional fluidized bed has been already given by Brandani & Foscolo in their very recent publication (1994).

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